LAMT 2026, May 17, 2026 - Guts Round Set 1

Team Name: ______________________________ Team ID: __________

1. **[9]** Find the number of zeroes in the decimal representation of

$$\frac{1111111111111111}{10001 \cdot 101 \cdot 11},$$

(where there are 16 ones in the numerator).

2. **[9]** Two real numbers b and c are chosen independently and uniformly at random from the interval $[0, 2]$. Find the probability that the quadratic $x^2 + bx - c^2 + 1 = 0$ has at least one real solution for x.

3. **[9]** A right triangle has integer leg lengths, and a hypotenuse of length $2\sqrt{2026}$. Find the sum of the lengths of the legs.

LAMT 2026, May 17, 2026 - Guts Round Set 2

Team Name: ________________________________ Team ID: ____________

4. **[10]** Let $f(n)$ denote the number of letters in the standard American spelling of n. For example $f(3) = 5$ since "three" has 5 letters in it. Find

$$f(f(f(f(1)))) + f(f(f(f(2)))) + \cdots + f(f(f(f(20)))).$$

5. **[10]** There exists a unique ordered pair (a, b) of positive integers satisfying the following: $a < b$, $a + b = 801$, and the second largest divisor of a plus the second largest divisor of b is 337. Find the ordered pair (a, b).

6. **[10]** Let $P(x)$ be a polynomial with nonnegative integer coefficients for which $P(1) = 10$ and $P(10) = 2026$. Find $P(2)$.

LAMT 2026, May 17, 2026 - Guts Round Set 3

Team Name: ______________________________ Team ID: __________

7. **[12]** Two chords of a circle with center O have lengths 14 and 16, are perpendicular, and intersect at A. The perpendicular bisector of OA meets the circle at two points B and C such that $BC = 17$. Find the radius of the circle.

8. **[12]** There is a unique polynomial $P(x)$ such that for every positive integer i,

$$P(i) = \sum_{j=1}^{i} \sum_{k=1}^{j} ijk.$$

Find the value of $P(-3)$.

9. **[12]** Let $x^3 - 9x^2 + 5x - 1$ have roots a, b, and c. Find

$$\left(\frac{1}{a} + \frac{1}{b} + 1 + c\right)\left(\frac{1}{b} + \frac{1}{c} + 1 + a\right)\left(\frac{1}{c} + \frac{1}{a} + 1 + b\right).$$

LAMT 2026, May 17, 2026 - Guts Round Set 4

Team Name: ______________________________ Team ID: __________

10. **[14]** Find the unique two digit positive integer x satisfying the following: x^2 is the four-digit number $\underline{a}\,\underline{b}\,\underline{c}\,\underline{d}$, and the two-digit numbers $\underline{a}\,\underline{b}$ and $\underline{c}\,\underline{d}$ sum to 121.

11. **[14]** Andrew draws positive integers at most 100 uniformly at random (without replacement), stopping once he has drawn all the multiples of 6. Find the probability he draws all the multiples of 5.

12. **[14]** Let $ABCD$ be a trapezoid with $AB \parallel CD$, $AB = 5$, $BC = 6$, and $CD = 10$. Let the midpoint of AB be M, and suppose $\angle CMD = 90^\circ$. Let AC and BD intersect at P. Find PM.

. . . .

LAMT 2026, May 17, 2026 - Guts Round Set 5

Team Name: ________________________________ Team ID: ____________

13. **[17]** Let $ABCDE$ be a regular pentagon and $DEFGHI$ a regular hexagon which don't overlap. The angle bisector of $\angle ICE$ intersects the perpendicular bisector of IH at X. Find $\angle AXE$, in degrees.

14. **[17]** Find the number of ordered triples (a, b, c) of integers in $\{0, 1, 2, \ldots, 25\}$ for which

$$20 \cdot 26 \leq a + 20b + 26c < 26^2.$$

15. **[17]** Let a_1, a_2, $\ldots$, a_{16} be positive integers less than or equal to $17 \cdot 19$. Suppose for each r in $\{1, 2 \ldots, 16\}$, there exist m and n in $\{1, 2, \ldots, 16\}$ for which 17 divides $a_m - r$ and 19 divides $a_n - r$. Find the maximum possible value of $a_1 + a_2 + \cdots + a_{16}$.

. . . .

LAMT 2026, May 17, 2026 - Guts Round Set 6

Team Name: ______________________________ Team ID: __________

16. **[18]** Consider the monic polynomial $f(x)$ with minimum degree satisfying

$$\frac{f(0)}{f(1)} = \frac{f(2)}{f(3)} = \frac{f(4)}{f(5)} = \frac{1}{2}.$$

Find $f(6)$.

17. **[18]** Over all integers $k \geq 1$, find the number of tuples $(a_1, a_2, \dots a_k, b_1, b_2, \dots b_k)$ of positive integers satisfying $a_1 = 1$, $a_{i+1} = a_i(b_i + 1)$, and

$$\sum_{i=1}^{k} a_i b_i = 383.$$

18. **[18]** Call an n digit positive integer $\underline{a_1 a_2 a_3 \dots a_n}$ *six-seven divisible* if there exists a positive integer $k < n$ such that 6 divides $\underline{a_1 a_2 a_3 \dots a_k}$ and 7 divides $\underline{a_{k+1} a_{k+2} \dots a_n}$. For example, 628 and 720 are six-seven divisible (6 divides both 6 and 72, and 7 divides both 28 and 0), while 362 and 90 are not. Find the number of four-digit six-seven divisible positive integers with a nonzero leading digit.

LAMT 2026, May 17, 2026 - Guts Round Set 7

Team Name: ________________________________ Team ID: ____________

19. **[21]** The numbers 1, 2, 3, and 4 are placed in a 4×4 grid such that each cell has exactly one number, and each number appears exactly 4 times. Two cells are called *friends* if they have the same entry and are in the same row or column. Find the number of arrangements for which each cell has exactly two friends.

20. **[21]** Find the smallest positive integer n for which $n \equiv 100 \pmod{101^2}$ and $n \equiv 101 \pmod{100^2}$.

21. **[21]** Let $ABCDE$ be a pentagon with $\angle A > 180°$ and $\angle B = \angle C = \angle D = \angle E$. The angle bisector of $\angle A$ meets CD at X. Given $BC = 13$, $DE = 11$, $CX = 12$, and $XD = 6$, find $\frac{AB}{AE}$.

LAMT 2026, May 17, 2026 - Guts Round Set 8

Team Name: ________________________________ Team ID: ____________

22. **[24]** Let $\lfloor x \rfloor$ denote the greatest integer less than or equal to x. Find the number of positive integers which can be expressed in the form
$$\lfloor \alpha \rfloor + \lfloor \alpha^2 \rfloor + \cdots + \lfloor \alpha^{10} \rfloor$$
for some real number $1 < \alpha \leq 2$.

23. **[24]** There are 12 cards, each with a number from 1 to 6 so that each number appears twice. Yibo draws 6 of these cards uniformly at random, and gives the other 6 to Gautham. Starting with Yibo, each player plays the card in their hand with the lowest value that is at least as large as all previously played cards. The game ends once a player cannot play a card. Find the probability that exactly 6 cards are played.

24. **[24]** Let ABC be a triangle with M the midpoint of BC. A point P lies inside ABC with $\angle APM = 90^\circ$ and $BP \perp AC$. Suppose $BP = 3$, $PM = 8$, and $AC = 21$. Find AM.

. . . .

Team Name: ________________________________ Team ID: ____________

25. **[25]** Let N be the number of 20-digit strings (each digit from 0 to 9) such that no two adjacent digits differ by more than 1, and the first digit is 5. Estimate $\log_{10}(N)$.

    Submit a number E. If the true answer is A, you will receive $\max(0, \lfloor 26 - 500|E - A| \rfloor)$ points.

26. **[25]** Positive reals $p_1, p_2, \ldots, p_{67}$ are randomly and independently generated such that, for each $n \in 1, 2, \ldots, 67$, the probability that $p_n = p$ for $p \in [1, \infty)$ is proportional to $\frac{7^7}{6!}(\ln(p))^6 p^{-8}$, and 0 for $p < 1$. Estimate the expected value of

$$\sum_{k=1}^{67} \log_{10}(p_k).$$

    Submit a number E. If the true answer is A, you will receive $\min\left(25, \left\lfloor 25e^{-0.67|A-E|} \right\rfloor\right)$ points.

27. **[25]** Let $\tau(n)$ denote the number of positive divisors of n. Estimate the value of

$$\sum_{n=1}^{10^6} \frac{\tau(n)}{n}.$$

    Submit a number E. If the true answer is A, you will receive $\max(0, \lfloor 26 - 4|E - A| \rfloor)$ points.

. . . .